Mirror-mist transition in brittle fracture

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A consistent explanation for the abrupt appearance of the mist zone in high velocity brittle fractures is given. The dependence of the boundary on flaw sizes is calculated. For low velocity fractures a gradual mirror-mist transition is demonstrated.

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I. INTRODUCTION

The morphological features of brittle fracture have been known for many years (see, e.g., [1]). Thus, on the surface of a fractured brittle specimen, the following characteristics are observed. The initial flaw is followed by the "mirror" plane of specular attribute; the mirror is often succeeded by the "mist" zone which consists of a multitude of secondary cracks. Finally, the "hackle" and the bifurcation zones appear in turn. There apparently exists a "background process" which causes the "roughness" of the cracked new surfaces to increase in a fractal manner (see, e.g., [2], but compare our papers [3–5] on a steady magnification at least across the mist zone). Another approach was the interaction of shear waves and cracks [6]. However, the major transitions, namely mirror-mist, mist-hackle, and hackle-branching are not described by this interaction. Despite the intimate descriptive knowledge, the basic understanding of these features is only partial. A physical explanation has been suggested [3–5] for the mist zone and for the mist-hackle transition. An interesting attempt using "hyper-elasticity" has recently been tried [7] in order to understand these transitions.

We present here a simple explanation for the mirror-mist transition, based on the shielding of the crack tip volume by a dynamic "process zone" created by the rise in temperature there. The main questions we wish to address are the conditions for the appearance of an "abrupt" (i.e., of narrow width) onset of the mist (and therefore leading to a clear mirror-mist boundary) in contrast to the appearance of a gradual (i.e., of extended width) mirror-mist transition.

II. MODEL

During the evolution of a fracture, the stress field in the vicinity of its tip changes [8]. All components of this stress behave like $Af(\vartheta,\upsilon)r^{-1/2}$, where A increases with the square root of the crack length, $\sqrt{c}$; $f(\theta,\nu)$ is a function of the fracture velocity ν and of the angle θ between the crack direction and the point where the stress is being evaluated, and r is the distance from this point to the crack tip. As was discussed previously [5], this stress can induce "visible" secondary cracks to develop from existing flaws in the material, under the two following (somewhat contradictory) conditions: (A) The stress intensity factor (SIF) at the front side of the flaw (Fig. 1) is large enough (for this to happen the flaw should be located close enough to the tip of the primary fracture) for it to be able to grow, namely, the SIF should be higher than the critical SIF (K_{Ic}) there. (B) The flaw must be located far enough from the tip in order to turn into a secondary crack of an observable length. This flaw starts to grow as a secondary crack until overtaken by the faster primary facture. Its final length thus depends on the flaw to tip distance. There is a major difference between the two conditions. While no secondary crack would develop at all if A is not fulfilled, B controls only its final magnitude. Since condition A is fulfilled immediately at the primary crack incipience, it could be concluded that secondary cracks would appear from the beginning of and all across the mirror area. These cracks, of gradually increasing magnitudes due to condition B, could be observed with proper magnification. At least they should be observed above some primary crack radius, much smaller than the mirror radius, when their magnitudes exceed the wavelength of visible light. In most cases, however, no secondary cracks are seen below a certain boundary, and beyond this boundary (the mirror radius) a sharp "onset" of secondary cracks is observed both with the naked eye and with the aid of a microscope. We assume that this phenomenon is due to a "dynamic" plastic (process) zone which develops around the tip of the propagating primary crack along its way and "shields" the flaws from its stress.

The feasibility of the shielding assumption will now be discussed and its implications analyzed. As is well-known, the static process zone in brittle materials is very small. For glass, for instance, the radius of this zone is estimated to be [9] around 5 Å. Such radii are too small to cause any noticeable shielding effects and in any case are much smaller than visible wavelengths. As stated above, we assume that the increase in shielding radius is caused by the elevated tem-

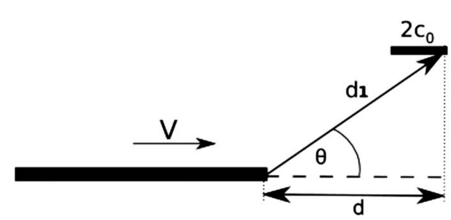

FIG. 1. Diagram of the primary crack (thick line) moving with a velocity V, and a secondary crack of initial length $2c_0$, that grows until overtaken by the primary one. The distance between the primary and secondary cracks is d_1 and its projection on the x axis is d.

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peratures surrounding the tip in high velocity fracture propagations. Such elevated temperatures (temperature increases of the order of 4000 °C) are both theoretically predicted (e.g., [10,11]) and experimentally measured (e.g., [12–14], and references therein) for brittle and ductile materials. The heating is caused by the transformation of the excess elastic energy into heat, and appears as a "high temperature" zone in which yielding and fracture properties are temporarily changed. The actual shape of the high temperature zone and its time evolution during (and following) the fracture process is not exactly known. The theoretical calculations are based on two different approaches. The first (e.g., [10]) is based on diffusion while the second (e.g., [11]) uses methods of heat waves. Both give somewhat dissimilar outcomes, but agree on some basic results, for instance, the direct relationship between crack velocity and heating. We shall therefore assume that a heated zone (HZ) accompanies the crack tip. It extends both behind and ahead of the tip. If a secondary crack is being formed within this zone, it would be unable to grow (see above) and would thus disappear. Although the HZ size increases slightly with time (as the crack velocity is increased up to a final velocity), since by condition A the distance from a susceptible "secondary flaw" to the primary fracture tip increases too, and even faster than that of the HZ, eventually the secondary cracks will be able to propagate, which will mark the onset of the "mist."

Thus, according to condition A above [3], for the secondary flaw to start growing it is needed that the SIF at its tip (Fig. 1), $\sigma_1(\pi c_0)^{1/2}$, be larger than the critical SIF (K_{Ic}) of the material, where σ_1 is the (high) principal stress at the flaw tip created by the primary fracture and c_0 is the "radius" of the flaw. According to [15], $\sigma_1 = K_I f(\theta,v)(2\pi d_1)^{-1/2}$. Here d_1 is the distance mentioned above (Fig. 1), K_I is the SIF of the primary fracture (which is proportional to $\sigma\sqrt{R}$ where σ is the remote stress and R is the primary fracture length), and $f(\theta,\nu)$ is a geometric function depending on the primary fracture velocity and the angle θ (Fig. 1). Condition A for mode I fracture becomes therefore

$$d \leqslant c_0\left(\frac{K_I}{K_{\mathrm{Ic}}}\right)^2 \frac{f^2(\theta,v)\cos\theta}{2}, \tag{1}$$

where d is the projection of d_1 on the primary crack plane and K_{Ic} is the critical SIF. As seen from Eq. (1) with the equality sign, the largest d (denoted d_m) for which a secondary crack can develop increases linearly with the primary fracture length. Since an estimate of the growth rule for the HZ is not easy to come by (see the references cited above) we shall assume that, for a fracture moving with high enough velocity, at the onset of the "mist" the HZ radius equals d_m and that following this instance the HZ radius will always be smaller than d_m. For the case of fracture under a constant loading stress $(K_I/K_{\mathrm{Ic}})^2 = R/c_c$, where c_c is the critical radius of the primary fracture at which it started to grow. We therefore postulate that the condition of appearance of the mist zone is that the HZ radius becomes equal to d_m of Eq. (1) with the equality sign. Denoting by R_M the primary fracture length at the mirror-mist transition (the so-called mirror radius) we get from Eq. (1)

$$R_M \cong \frac{Q}{c_0}, \tag{2}$$

where $Q = 2r_p c_c/(f^2 \cos\vartheta)$ is a constant for a specific failure experiment and r_p is a measure of the HZ radius at the transition. Equation (2) implies that the onset of the mist should not occur simultaneously for all flaws. According to measurements [16] however, the length distribution function of flaws is sharply peaked. It can be assigned a probability distribution function (PDF) of the form [17]

$$p(c_0) = A_1 c_0^{-n} \exp(-c_{sc}/c_0), \tag{3}$$

where $A_1 = c_{sc}^{n-1}/\Gamma(n-1)$ is the normalizing factor, c_{sc} is a parameter (in fact the peak of the distribution occurs at $c_0 = c_{sc}/n$), Γ is the gamma function, and n is a constant exponent. These constants are estimated to be [18] $n = 7.5$ and $c_{sc} = 11.7$ μm for glass.

Note that these numbers pertain to surface flaws. For bulk flaws which are the ones considered here, c_{sc} is much smaller. From Eqs. (2) and (3), the PDF for the mirror radii should be (see, e.g., [19])

$$p(R_M) = A_2 R_M^{n-2} \exp(-R_M/R_0), \tag{4}$$

where $A_2 = A_1 Q^{-n+1}$ is a normalizing factor and $R_0 = Q/c_{sc}$. This PDF is again peaked, so the onset of the mist is "quite" abrupt with occasional appearances of secondary cracks from exceptionally large flaws in the mirror zone. The peak in R_M occurs for

$$R_M^{\mathrm{peak}} = (n-2)R_0 = 2(n-2)r_p c_c/(f^2 c_{sc} \cos\theta) \tag{5}$$

and equating R_M^{peak} with the mirror-mist boundary, r_p can be calculated from Eq. (5).

III. RESULTS AND DISCUSSION

As an example consider the experimental results of Ref. [20] for which, at R_M, $K_I/K_{\mathrm{Ic}} \sim 2.68$. $f(\theta,v) \approx 1.4$ and $\theta \approx 60°$ [21]. This implies, for a peaked c_0 at 1.5 μm, an r_p of 7.5 μm, which is, of course, much larger than the static r_p. The flaw, of "radius" c_0, elongates until overtaken by the primary fracture which moves at a faster velocity [3]. The much longer secondary crack thus created is the one observed in the mist initiation. For $K_I/K_{\mathrm{Ic}} = 2.68$ the magnification [3] of the length of the flaw (to get the length of the secondary crack) at the mist onset is ~60 which means that these secondary cracks, of lengths of the order of 0.1 mm, are indeed immediately visible.

The fact that the mist starts at already quite high magnifications can he responsible for the relative deficiency of "circlelike" secondary crack shapes in fast fracture [4] since these shapes were predicted to appear for low magnifications only.

We consider now the question whether the mirror-mist transition is always an abrupt one. If our assumption (shielding by temperature) regarding its origin is true, we would expect to have *no* discernable mirror-mist boundary for low-velocity fractures, where (see the above mentioned references on temperature rise in materials) the temperature in-

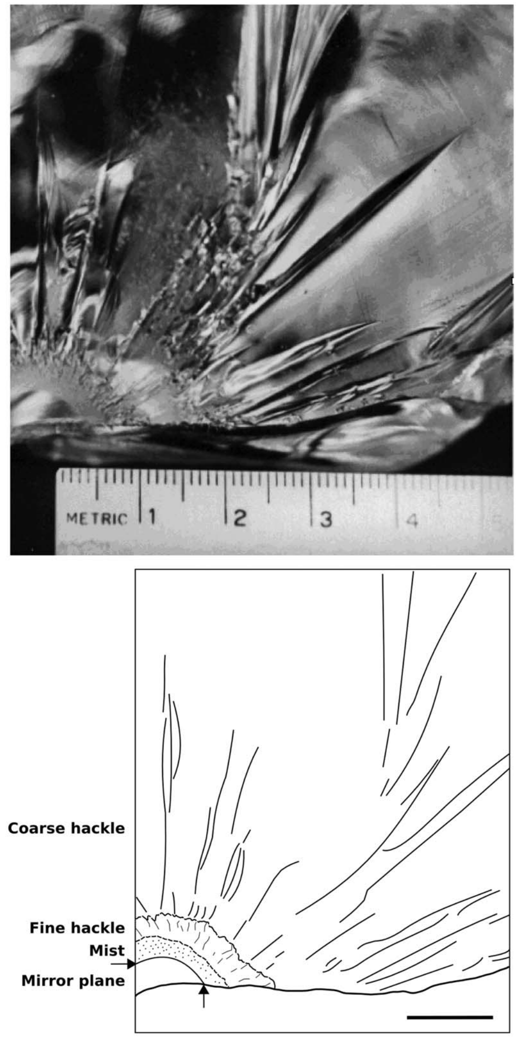

FIG. 2. An abrupt transition from the mirror plane to the mist zone in a glass fractured rapidly by impact. The classic sequence of fractographic elements appears: mirror plane, mist zone, narrow zone of fine hackles, and the coarse hackles (the latter extends throughout the fracture surface). The boundary between the mirror plane and mist zone is shown in the inset by vertical and horizontal arrows.

crease should be small and the shielding effect weak or nonexistent. To check this inference, we use our previous experimental results. Figure 2 shows a soda lime glass fractured rapidly by high stress, induced by an impacting hammer. The abrupt transition between the mirror plane and mist can be seen along a thin line (vertical and horizontal arrows in inset). The specular nature of the mirror plane stands out next to the uniform microcracking in the mist zone, which is also separated well from the hackle zone. On the other hand,

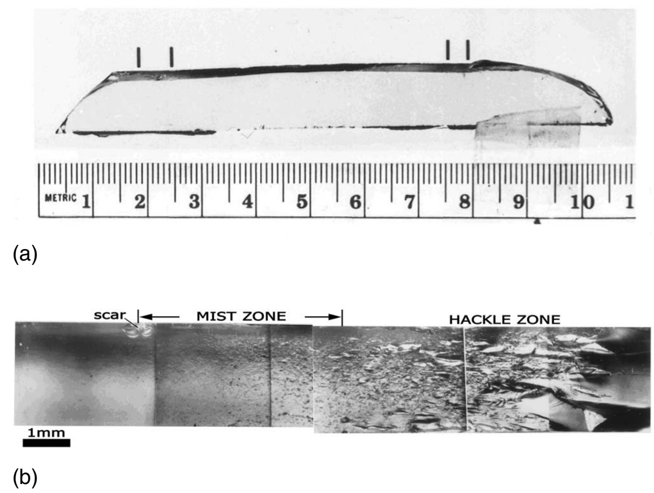

FIG. 3. (a) Large mirror plane that formed by a slow fracture. The two inner vertical lines designate the "boundaries" between the mirror plane and the mist zone, on both sides of the mirror. The two outer vertical lines mark the respective boundaries between the mist and fine hackle zones. (b) Steady growth of the microcracks from the end of the mirror plane (around the accidental scar) to the hackle zone, showing a heterogeneous mist zone that gradually transforms between these two zones.

a gradual mirror-mist transition is demonstrated in a soda lime glass bottle, when fractured by low internal pressure in a hot bath [22,23].

Figure 3(a) shows the long radius of the mirror plane of the latter ($\sim$25 mm), compared to the short radius shown in Fig. 2 ($\sim$7 mm). Recall [24] that the length of the mirror plane R_M is inversely correlative with the fracture stress σ according to $A=\sigma R_M^{1/2}$, with A a constant). Consequently, the transition from the mirror plane to the mist zone is not abrupt, but rather gradual [Fig. 3(b)]. In fact, there is a gradual growth of visible microcracks from around the accidental scar to the hackle zone.

Finally, regarding shielding, there is yet no exact physical understanding of this phenomenon. A possible explanation is that in the heated zone, due to thermal expansion, secondary cracks (flaws) get blunted thus increasing K_{Ic}. A different possibility is that some strain is accommodated by thermal expansion thus lowering the strain which arises from the existing stress thereby lowering the local stress, rendering K_I at the secondary crack tip to be below K_{Ic}.

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